Steady-State Sin

- **Sine Eqns:**

$$f = \frac{1}{T} \quad T = \frac{2\pi}{\omega} \quad \omega = 2\pi f$$

- **Leading/Lagging:** $v(t) = V_m \sin(\omega t + \theta)$

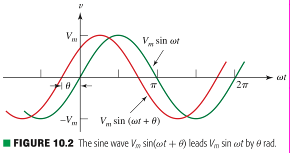


FIGURE 10.2 The sine wave $V_m \sin(\omega t + \theta)$ leads $V_m \sin \omega t$ by θ rad.

$\sin \omega t$ as ***lagging*** $\sin(\omega t + \theta)$ by θ rad
leading $\sin(\omega t + \theta)$ by $-\theta$ rad
or as leading $\sin(\omega t - \theta)$ by θ rad

- **Convert Sin-to-Cos:**

$$\sin(\omega t \pm 90°) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90°) = \mp\sin\omega t$$

Real Power:

- **Eqns:**

$P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$

The real power P is the average power in watts delivered to a load;

Dot Convention:

Current entering dot, (+) @ 2nd coil dot

Current entering undot, (+) @ 2nd coil undot

Reactive Factor:

VAR

- **Eqns:**

$$Q = I_m\{S\} = \frac{1}{2}V_m I_m \sin\theta = V_{rms} I_{rms} \sin\theta$$

$Q = 0$ for resistive loads (unity pf).
$Q < 0$ for capacitive loads (leading pf).
$Q > 0$ for inductive loads (lagging pf).

Impedance/Admittance:

Element	Time domain	Frequency domain	Impedance	Admittance
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

- **Admittance:**

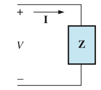
$$\mathbf{Y} = G + jB = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX} \qquad G = \textit{conductance (s)}, \quad B = \textit{susceptance}$$

- Phasor analysis can only be performed on single-frequency circuits. Otherwise, superposition must be invoked, and the *time-domain* partial responses added to obtain the complete response.

Apparent Power: VA

- **Eqns:**

$$S = |\mathbf{S}| = |\mathbf{V}_{rms}|\,|\mathbf{I}_{rms}| = \sqrt{P^2 + Q^2}$$



$|\mathbf{S}| = V_{\text{eff}} I_{\text{eff}}$, and is the maximum value the average power can be; $P = |\mathbf{S}|$ only for purely resistive loads.

Complex Power: VA

- **Eqns:**

The reactive power Q is a measure of the energy exchange between the source and the reactive part of the load.

$$S = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = |\mathbf{I}_{rms}|^2 Z = \frac{|\mathbf{V}_{rms}|^2}{Z^*}$$

The real power P is the average power in watts delivered to a load;

$$\mathbf{S} = I^2_{rms}(R + jX) = P + jQ$$
$$P = I^2_{rms}R = \frac{1}{2}I^2_m R = V_{rms}I_{rms}\cos(\theta_v - \theta_i)$$
$$Q = I^2_{rms}X = \frac{1}{2}I^2_m X = V_{rms}I_{rms}\sin(\theta_v - \theta_i)$$

Average Power:

In the sinusoidal steady state, $P = \frac{1}{2}V_m I_m \cos(\theta - \phi)$, where θ is the angle of the voltage and ϕ is the angle of the current. Reactances do not contribute to P.

- The instantaneous power absorbed by an element is given by the expression $p(t) = v(t)i(t)$.
- The average power delivered to an impedance by a sinusoidal source is $\frac{1}{2}V_m I_m \cos(\theta - \phi)$

- **Eqns:**

The average power

$$\text{P} = \frac{1}{\text{T}}\int_0^{\text{T}} \text{p(t) dt} = \frac{1}{2}\text{Re}\{\text{VI}^*\} = \frac{1}{2}\text{V}_m\text{I}_m\cos(\theta)$$

The average power delivered

$$P = V_{\text{eff}} I_{\text{eff}} \cos(\theta - \phi)$$

When $\theta_v = \theta_i$, implies a purely resistive circuit

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I^2_m R = \frac{1}{2}|\mathbf{I}|^2 R$$

the average power absorbed by a resistor R

$$P = I^2_{\text{rms}}R = \frac{V^2_{\text{rms}}}{R}$$ always absorbs power.

Power Factor:

- **Eqns:**

$$PF = \frac{P_{Avg}}{|S|} = \cos\theta = \frac{P_{Avg}}{V_{rms}I_{rms}}$$

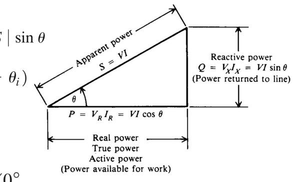
(+) Q: θ-$\phi > 0 \rightarrow$ Lagging (L)

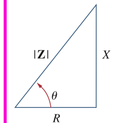
(-) Q: θ-$\phi < 0 \rightarrow$ Leading (C)

Ratio of the average power to the apparent power. The PF is unity for a purely resistive load, and zero for a purely reactive load.

$P = |S|\cos\theta \quad Q = |S|\sin\theta$

$$\tan\theta = \frac{Q}{P} = \tan(\theta_v - \theta_i)$$





$$\mathbf{P} = P\angle 0°$$
$$\mathbf{Q}_L = Q_L\angle 90° \qquad \mathbf{Q}_C = Q_C\angle -90°$$

The load impedance

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m\angle\theta_v}{I_m\angle\theta_i} = \frac{V_m}{I_m}\angle\theta_v - \theta_i \qquad \mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = \frac{V_{rms}}{I_{rms}}\angle\theta_v - \theta_i$$

Phasor:

- **Gen. Form:** $\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2} \qquad R = |\mathbf{Z}|\cos\theta$$
$$\theta = \tan^{-1}\frac{X}{R} \qquad X = |\mathbf{Z}|\sin\theta$$

Time domain	Phasor domain
$V_m\cos(\omega t + \phi)$	$V_m\angle\phi$
$V_m\sin(\omega t + \phi)$	$V_m\angle\phi - 90°$

Y-Connected:

- **Phase Volt:** $V_L = \sqrt{3}V_p$

$$V_{AN} = V_p(0°)$$
$$V_{BN} = V_p(-120°)$$
$$V_{CN} = V_p(-240°)$$

- **Line Volt:**

$$V_{AB} = V_{ab} = \sqrt{3}V_p(30°)$$
$$V_{BC} = V_{bc} = \sqrt{3}V_p(-90°)$$
$$V_{CA} = V_{ca} = \sqrt{3}V_p(-210°)$$

- **Line/Phase Current:**

$$I_{aA} = I_{AN} = \frac{V_{AN}}{Z_p}$$
$$I_{bB} = I_{BN} = \frac{V_{BN}}{Z_p}$$
$$I_{cC} = I_{CN} = \frac{V_{CN}}{Z_p}$$

- **Power Per Phase:**

$$P_p = \frac{V_L}{\sqrt{3}}I_L\cos\theta$$

Δ-connected:

- **Phase/Line Volt:**

$$V_{AB} = V_{ab} = \sqrt{3}V_p(30°)$$
$$V_{BC} = V_{bc} = \sqrt{3}V_p(-90°)$$
$$V_{CA} = V_{ca} = \sqrt{3}V_p(-210°)$$

- **Phase Current:** $V_L = \sqrt{3}V_p$, $I_L = \sqrt{3}I_p$

$$I_{AB} = \frac{V_{AB}}{Z_p} \quad I_{BC} = \frac{V_{BC}}{Z_p} \quad I_{CA} = \frac{V_{CA}}{Z_p} \qquad Z_Y = \frac{Z_\Delta}{3}$$

- **Line Current:**

$$I_{aA} = (\sqrt{3}(-30°))\frac{V_{AB}}{Z_p}$$
$$I_{bB} = (\sqrt{3}(-30°))\frac{V_{BC}}{Z_p}$$
$$I_{cC} = (\sqrt{3}(-30°))\frac{V_{CA}}{Z_p}$$

- **Power Per Phase:**

$$P_p = V_L\frac{I_L}{\sqrt{3}}\cos\theta$$

$$P_{\text{T}} = \sqrt{3}V_{\text{L}}I_{\text{L}}\cos\theta$$

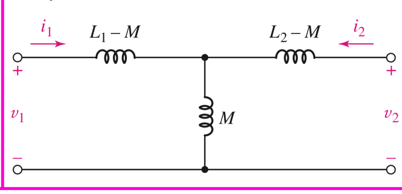
Ideal Transformer:

- **Eqns:**

$$a = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1}$$
$$\mathbf{V}_1 = \mathbf{I}_1\mathbf{Z}_{\text{in}} = \mathbf{I}_1\frac{\mathbf{Z}_L}{a^2}$$
$$\mathbf{V}_2 = \mathbf{I}_2\mathbf{Z}_L \qquad a = \frac{\mathbf{V}_2}{\mathbf{V}_1}$$
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = a^2\frac{\mathbf{I}_2}{\mathbf{I}_1} \qquad [Z_L \neq 0]$$

If $a > 1$, ***step-up transformer.***
If $a < 1$, ***step-down transformer.***

T-Equivalent:



Mag Scaling:

- **Eqns:**

$$R' = K_m R$$
$$L' = K_m L = K_m\mathbf{Z}_L = j\omega K_m L$$
$$C' = \frac{C}{K_m} = K_m\mathbf{Z}_C = \frac{1}{j\omega C/K_m}$$

the series or parallel RLC circuit

$$\omega' = \omega$$
$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_m LC/K_m}} = \frac{1}{\sqrt{LC}} = \omega_0$$

Complex Freq:

- **Gen Form for a real Sine Volt:**

$$v(t) = \mathbf{K}_1 e^{\mathbf{s}_1 t} + \mathbf{K}_2 e^{\mathbf{s}_2 t}$$

where

$$\mathbf{K}_1 = \tfrac{1}{2}V_m e^{j\theta} \quad \mathbf{s}_2 = -j\omega$$
$$\mathbf{K}_2 = \mathbf{K}_1^* \quad \mathbf{s}_2 = \mathbf{s}_1^*$$

- **Pure Re() expo damped sine Fn:**

$$v(t) = V_m e^{\sigma t}\cos(\omega t + \theta)$$
$$i(t) = I_m e^{\sigma t}\cos(\omega t + \phi)$$

- **Forcing Fn:**

$$v(t) = Ve^{st} = V_m\angle\theta e^{st}$$
$$i(t) = Ie^{st} = I_m\angle\theta$$

- The complex frequency $\mathbf{s} = \sigma + j\omega$ is the general case; dc ($\mathbf{s} = 0$), exponential ($\omega = 0$), and sinusoidal ($\sigma = 0$) functions are special cases.

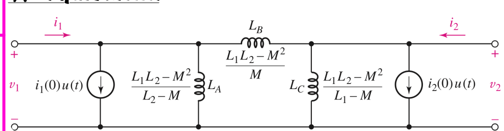
Freq Scaling:

- **Eqns:**

$$R' \rightarrow R \qquad L' \rightarrow \frac{L}{K_f} = j(\omega K_f)L' = j\omega L$$
$$C' \rightarrow \frac{C}{K_f} = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C}$$
$$\omega' = K_f\omega$$
$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(L/K_f)(C/K_f)}} = \frac{K_f}{\sqrt{LC}} = K_f\omega_0$$

for the bandwidth $B' = K_f B$ — the quality factor $Q' = Q$

π-Equivalent:



Induction:

$$PF_{load} = \sqrt{3}V_L I_L\cos\theta$$

- **Energy Stored:**

$$w(t) = \tfrac{1}{2}L_1[i_1(t)]^2 + \tfrac{1}{2}L_2[i_2(t)]^2 \pm M[i_1(t)][i_2(t)]$$
$$W = \tfrac{1}{2}L_1 I_1^2 + \tfrac{1}{2}L_2 I_2^2 + MI_1I_2$$

If one current enters a dot-marked terminal while the other leaves a dot-marked terminal, the sign of the mutual energy term is reversed.

- **Coupling Coefficient:**

$$k = \frac{M}{\sqrt{L_1L_2}}, \quad \text{Since } M \leq \sqrt{L_1L_2}, \quad 0 \leq k \leq 1$$

M approaches its maximum value

Laplace Trans Ops:

Property	$f(t)$	$F(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_0^t f(x)dx$	$\frac{1}{s}F(s)$

AC Pwr Eqns:

Power	General expression	R	L	C
Instantaneous power	$p = VI\cos\varphi\,(1 - \cos 2\omega t) + VI\sin\varphi\,\sin 2\omega t$	$p_R = VI\,(1 - \cos 2\omega t)$	$p_L = VI\sin 2\omega t$	$p_C = -VI\sin 2\omega t$
Active power	$p = VI\cos\varphi$	$P_R = VI = \frac{1}{2}V_m I_m = I^2R = \frac{V^2}{R}$	$P_L = 0$	$P_C = 0$
Reactive power	$Q = VI\sin\varphi$	$Q_R = 0$	$Q_L = VI = I^2X_L = \frac{V^2}{X_L}$	$Q_C = -VI = -I^2X_C = -\frac{V^2}{X_C}$
Apparent power	$S = IV = I^2Z = \frac{V^2}{Z}$			

Mag & Freq Scaling:

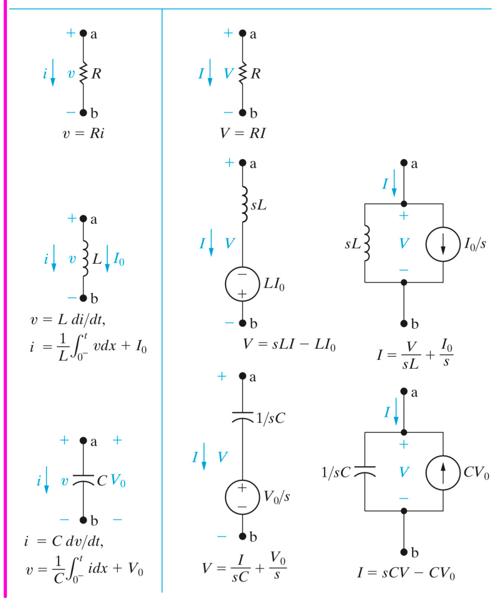
- **Eqns:** $R' = K_m R, \quad L' = \frac{K_m}{K_f}L$

$$C' = \frac{1}{K_m K_f}C, \quad \omega' = K_f\omega$$

Laplace Trans Pairs:

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s + a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s + a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s + a)^{n+1}}$
$\sin\omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos\omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2 + \omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s + a)^2 + \omega^2}$
$e^{-at}\cos\omega t$	$\frac{s + a}{(s + a)^2 + \omega^2}$

TIME DOMAIN — FREQUENCY DOMAIN



Resonances Ckt Eqns:

the ***damping factor:***

$$\zeta = \frac{\alpha}{\omega_0} = \frac{1}{2Q_0}$$

Parallel circuit

$$\alpha = \frac{\omega_0}{2Q_0} = \tfrac{1}{2}\mathcal{B}$$
$$Q_0 = 2\pi f_0 RC = \omega_0 RC = R\sqrt{\frac{C}{L}} = \frac{R}{|X_{C,0}|} = \frac{R}{|X_{L,0}|}$$

$v = (V\sqrt{2})\cos\omega t \qquad \mathbf{V}_{\text{eff}} = V\angle 0°$

$i = (I\sqrt{2})\cos(\omega t - \theta) \qquad \mathbf{I}_{\text{eff}} = I\angle -\theta$

$P = VI\cos\theta$, $Q = VI\sin\theta$ and $S = VI$ (apparent power)

	Z	i	$\mathbf{I}_{\text{eff}}$	$p(t)$	P	Q	S
R	R	$\frac{V\sqrt{2}}{R}\cos\omega t$	$\frac{V}{R}\angle 0°$	$\frac{V^2}{R}(1 + \cos 2\omega t)$	$\frac{V^2}{R}$	0	$\frac{V^2}{R}$
L	$jL\omega$	$\frac{V\sqrt{2}}{L\omega}\cos(\omega t - 90°)$	$\frac{V}{L\omega}\angle -90°$	$\frac{V^2}{L\omega}\sin 2\omega t$	0	$\frac{V^2}{L\omega}$	$\frac{V^2}{L\omega}$
C	$\frac{-j}{C\omega}$	$V\sqrt{2}C\omega\cos(\omega t + 90°)$	$VC\omega\angle 90°$	$-V^2C\omega\sin 2\omega t$	0	$-V^2C\omega$	$V^2C\omega$

Ex: Y-Y phase sys, given VL. Calc iL and Zp.

a) w/ Pt & PF

$$P_T = \sqrt{3}V_L I_L\cos\theta$$

solve for I_L

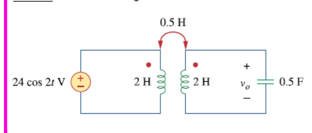
$V_{an} = V_L/\sqrt{3}$ $\quad \cos^{-1}(PF)$

$$Z_p = \frac{V_{an}}{I_{aA}} \qquad \boxed{Z_p = 7.23 + j4.48\ \Omega}$$

b) w/ Pp & PF

$$P_p = \frac{V_L}{\sqrt{3}}I_L\cos\theta \qquad \theta: (-) \text{ b/c lead}$$
$$Z_p = \frac{V_{an}}{I_{aA}}$$

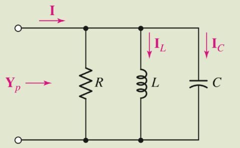
Ex: Find v_o

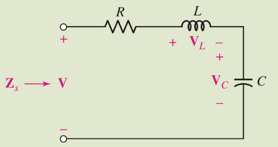


$$24 = j4I_1 - jI_2, \qquad I_2 = -j2.1818,$$
$$0 = -I_1 + 3I_2 \qquad V_o = -jI_2 = -2.1818$$

$\text{V}_o = \underline{\mathbf{-2.1818cos2t\ V}}$

Para/Series Ckt Summary:





$Q_0 = \omega_0 RC \qquad \alpha = \dfrac{1}{2RC}$

$|\mathbf{I}_L(j\omega_0)| = |\mathbf{I}_C(j\omega_0)| = Q_0|\mathbf{I}(j\omega_0)|$

$\mathbf{Y}_p = \dfrac{1}{R}\left[1 + jQ_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)\right]$

$Q_0 = \dfrac{\omega_0 L}{R} \qquad \alpha = \dfrac{R}{2L}$

$|\mathbf{V}_L(j\omega_0)| = |\mathbf{V}_C(j\omega_0)| = Q_0|\mathbf{V}(j\omega_0)|$

$\mathbf{Z}_s = R\left[1 + jQ_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)\right]$

Exact expressions

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1 - \left(\frac{1}{2Q_0}\right)^2}$$

$$\mathcal{B} = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0} = 2\alpha$$

Approximate expressions

$(Q_0 \geq 5 \qquad 0.9\omega_0 \leq \omega \leq 1.1\omega_0)$

$\omega_d \approx \omega_0$

$\omega_{1,2} \approx \omega_0 \mp \frac{1}{2}\mathcal{B}$

$\omega_0 \approx \frac{1}{2}(\omega_1 + \omega_2)$

$\mathbf{Y}_p \approx \dfrac{\sqrt{1+N^2}}{R}\underline{/\tan^{-1} N}$

$\mathbf{Z}_s \approx R\sqrt{1+N^2}\underline{/\tan^{-1} N}$

PARALLEL OR SERIES RLC CIRCUIT FREQUENCY RESPONSE RELATIONSHIPS

QUANTITY	SYMBOL	RELATIONSHIP
Resonant frequency (parallel or series)	ω_o	$1/\sqrt{LC}$ or $\sqrt{\omega_1\omega_2}$
Bandwidth	β	$1/RC$ (parallel) R/L (series)
Quality factor (parallel or series)	Q	ω_o/β $\omega_o RC$ (parallel) $\omega_o L/R$ (series)
Corner frequency (parallel or series)	ω_1	$\omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$
	ω_2	$\omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$

Characteristic		Series circuit	Parallel circuit
Resonant frequency, ω_0	$\omega_0 = \sqrt{\omega_1\omega_2}$	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q		$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$B = \omega_2 - \omega_1$	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2		$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2		$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

the instantaneous energy stored in the inductor is

$$w_L(t) = \frac{1}{2}Li_L^2 = \frac{1}{2L}\left[\frac{R\mathbf{I}_m}{\omega_0}\sin\omega_0 t\right]^2$$

The energy stored in the capacitor is

$$w_C(t) = \frac{1}{2}Cv^2 = \frac{\mathbf{I}_m^2 R^2 C}{2}\cos^2\omega_0 t$$

The total *instantaneous* stored energy is

$$w(t) = w_L(t) + w_C(t) = \frac{\mathbf{I}_m^2 R^2 C}{2}$$

Ex: Given: $V = 110\ V_{rms}$ $f = 60\ Hz$ $|S| = 120\ VA$ $PF = 0.707$ lag

(a) Calculate the complex power.
(b) Find the rms current supplied to the load.
(c) Determine $\mathbf{Z}$.
(d) Assuming that $\mathbf{Z} = R + j\omega L$, find the values of R and L.

(a) S=120, $pf = 0.707 = \cos\theta \longrightarrow \theta = 45°$

$S = S\cos\theta + jS\sin\theta = \underline{84.84 + j84.84\ \text{VA}}$

(b) $S = V_{rms}I_{rms}$

$I_{rms} = \dfrac{S}{V_{rms}} = \dfrac{120}{110} = \underline{1.091\ \text{A rms}}$

(c) $S = I_{rms}^2 Z$

$Z = \dfrac{S}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

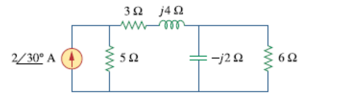
(d) If $Z = R + j\varpi L$, then $R = \underline{\mathbf{71.278\ \Omega}}$

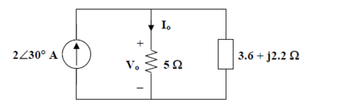
$\omega L = 2\pi f L = 71.278 \longrightarrow L = \dfrac{71.278}{2\pi x60} = \underline{0.1891\ \text{H}}$

Three-Phase Power:

Symbol	Quantity	Formula	Unit
P_P	Real power per phase	$P_P = V_P I_P \cos\theta$	Watt (W)
P_T	Total three-phase real power	$P_T = 3P_P = 3V_P I_P\cos\theta = \sqrt{3}V_L I_L\cos\theta$	Watt (W)
Q_P	Reactive power per phase	$Q_P = V_P I_P\sin\theta = \sqrt{S_P^2 - P_P^2}$	Volt-ampere-reactive (VAR)
Q_T	Total three-phase reactive power	$Q_T = 3Q_P = \sqrt{3}V_L I_L\sin\theta = \sqrt{S_T^2 - P_T^2}$	Volt-ampere-reactive (VAR)
S_P	Apparent power per phase	$S_P = V_P I_P$	Volt-ampere (VA)
S_T	Total three-phase apparent power	$S_T = 3S_P = 3V_P I_P = \sqrt{3}V_L I_L$	Volt-ampere (VA)
PF	Power factor	$\text{PF} = \cos\theta = \frac{p}{s}$	

Ex: $|S| = ?$ of source





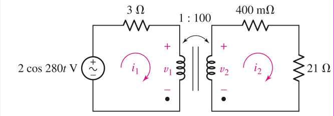
$\mathbf{I}_o = \dfrac{3.6 + j2.2}{8.6 + j2.2}(2\angle 30°) = 0.95\angle 47.08°$

$\mathbf{V}_o = 5\mathbf{I}_o = 4.75\angle 47.08°$

$\mathbf{S} = \frac{1}{2}\mathbf{V}_o\mathbf{I}_s^* = \frac{1}{2}\cdot(4.75\angle 47.08°)(2\angle -30°)$

$\mathbf{S} = 4.75\angle 17.08° = \mathbf{4.543 + j1.396\ VA}$

Ex: P_{avg} in Ω's = ?

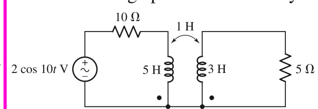


$Z_{in} = \dfrac{Z_L}{a^2} \qquad I_2 = \dfrac{I_1}{a}$

$I_1 = \dfrac{V_p}{Z_p + Z_{in}} \qquad P_{400\ m\Omega} = \dfrac{1}{2}R|I_2|^2$

Ex:

find the average power absorbed by



(*a*) the source;

$P_{source,avg} = \dfrac{V_S I_1}{2}\cos\theta = 9.167 \times 10^{-3}$ W

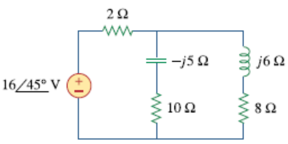
(*b*) each of the two resistors;

$P_{10\Omega,avg} = \dfrac{10\ I_{1,rms}^2}{2} \qquad P_{5\Omega,avg} = \dfrac{5\ I_{2,rms}^2}{2}$

(*c*) each of the two inductances:

$P_{5H,avg} = 0$ W

Ex:



(a) the power factor

$\mathbf{Z}_T = 2 + (10 - j5)\,||\,(8 + j6) = 8.188\angle 5.382°$

pf = cos(5.382°) = **0.9956** **(lagging)**

(b) the average power delivered by the source

$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \dfrac{|\mathbf{V}|^2}{2\mathbf{Z}^*} = \dfrac{(16)^2}{(2)(8.188\angle -5.382°)}$

$\mathbf{S} = 15.63\angle 5.382°$

$\mathbf{P} = \mathbf{S}\cos\theta = \mathbf{15.56\ W}$

(c) the reactive power

$\mathbf{Q} = \mathbf{S}\sin\theta = \mathbf{1.466\ VAR}$

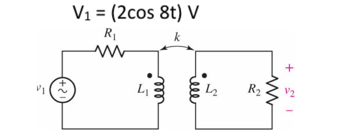
(d) the apparent power

$\mathbf{S} = |\mathbf{S}| = \mathbf{15.63\ VA}$

(e) the complex power

$\mathbf{S} = 15.63\angle 5.382° = \mathbf{15.56 + j1.466\ VA}$

Ex: L_1 = 4 mH, L_2 = 12 mH
R_1 = 1 ohms, R_2 = 10 ohms
V_1 = (2cos 8t) V



a. Find M if K = 0.6

$M = k\sqrt{L_1 L_2}$

b. Find i_1 and i_2

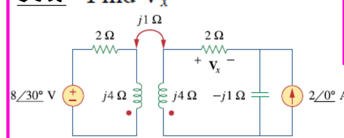
$2\angle 0 = i_1[1 + jwL_1] + i_2[jwM]$

$0 = i_2[10 + jwL_2] + i_1[jwM]$

c. Find V_2.

$V_2 = R_{10\Omega}i_2$

Ex: Find $\mathbf{V}_x$

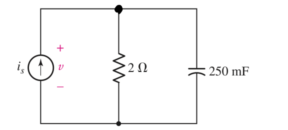


Steps

1) ST 2) Mesh

3) $\mathbf{V}_x = 2\mathbf{I}_2 = \underline{\mathbf{2.074\angle 21.12°}}$

Ex: $v_s(t) = ?$



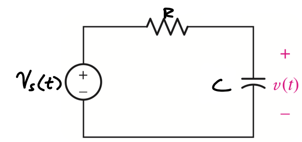
Given: $I = I_m\angle\phi$

$s = \sigma + j\omega$

$i_s(t) = \dfrac{v(t)}{2} + C\dfrac{dv(t)}{dt}$

$I = \dfrac{V}{2} + sCV \rightarrow V = \dfrac{I_m\angle\phi}{0.5 + sC}$

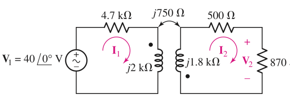
Ex: V(t)=? @t=0.8s



Steps

1) Nodal
2) Expand V(s)
3) Write V(t)
4) Plug in t=0.8s

Ex: calculate $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_2/\mathbf{V}_1$, and $\mathbf{I}_2/\mathbf{I}_1$.



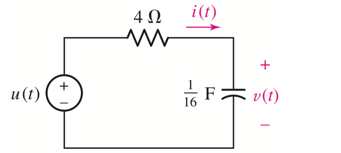
(a) $I_1 = 7.73\angle -20.3°$ mA

$I_2 = 2.549\angle -163.2°$ mA

$\dfrac{V_2}{V_1} = 0.055\angle -163.2°$

$\dfrac{I_2}{I_1} = 0.33\angle -142.9°$

Ex: **Determine $i(t)$ for $t > 0$**



$v(0^-) = 9$ V

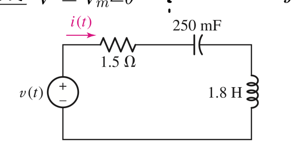
$$u(t) = 4i(t) + v(0^-) + 16\int_{0^-}^{t} i(t')\,dt'$$

$$\frac{1}{\mathbf{s}} = 4\mathbf{I(s)} + \frac{9}{\mathbf{s}} + \frac{16}{\mathbf{s}}\mathbf{I(s)}$$

$$\mathbf{I(s)} = -\frac{2}{\mathbf{s}+4}$$

$$i(t) = -2e^{-4t}u(t)\ \text{A}$$

Ex: $V = V_m\angle\theta$; $S = \sigma + j\omega$



$v(t) = Ve^{st} \longrightarrow V = V_m\angle\theta$

a) $S^* = \sigma - j\omega$

b) $v(t) = V_m e^{\sigma t}\cos(\omega t + \theta)$

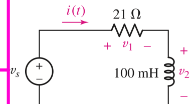
c) $i(t) = ?$

$v(t) = Ri(t) + \frac{1}{c}\int i(t)dt + i_0 + L\dfrac{di(t)}{dt}$

$V = RI + \dfrac{1}{sc}I + sLI$

$I = \dfrac{V_m\angle\theta}{R + \frac{1}{sC} + sL}\ \therefore I = I_m\angle\phi$

Ex: $\dfrac{V_2}{V_1} = ?$



Given: $S = \sigma + j\omega$

$\dfrac{V_2}{V_1} = \dfrac{V_L(t)}{V_R(t)_{st}} = \dfrac{LIse^{st}}{RIe^{st}} = \dfrac{sL}{R}$

Ex: Para RLC

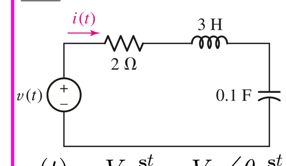
$R = 60\ \Omega$, $L = 1$ mH, and $C = 50\ \mu F$

$\omega_o = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3}x50x10^{-6}}} = \underline{4.472\ \text{krad/s}}$

$B = \dfrac{1}{RC} = \dfrac{1}{60x50x10^{-6}} = \underline{333.33\ \text{rad/s}}$

$Q = \dfrac{\omega_o}{B} = \dfrac{4472}{333.33} = \underline{13.42}$

Ex: $I_m = ?$ & $\phi = ?$



$v(t) = Ve^{st} = V_m\angle\theta e^{st}$

$v(t) = Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\int i\,dt$

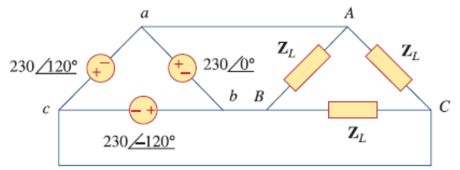
$60\underline{/10°} = 2\mathbf{I} + 3s\mathbf{I} + \dfrac{10}{\mathbf{s}}\mathbf{I}$

$\mathbf{I} = \dfrac{60\underline{/10°}}{2 + 3s + 10/s}$

Sub in s and solve for I then convert to time domain i(t)

$i(t) = 5.37e^{-2t}\cos(4t - 106.6°)$ A

Ex: Three 230-V generators form a delta-connected source $\mathbf{Z}_L = 10 + j8\ \Omega$ per phase



(a) Determine the value of $\mathbf{I}_{AC}$.

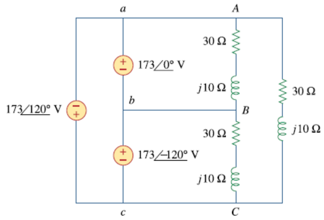
$I_{AC} = \dfrac{-230\angle 120°}{10 + j8} = \dfrac{-230\angle 120°}{12.806\angle 38.66°} = \underline{17.96\angle -98.66°}$ A(rms)

(b) What is the value of $\mathbf{I}_b$?

$I_{bB} = I_{BC} - I_{AB} = \dfrac{230\angle -120}{10 + j8} - \dfrac{230\angle 0°}{10 + j8}$

$= 31.10\angle 171.34°$ A

Ex: For the Δ-Δ circuit calculate the phase and line currents.



$$\mathbf{Z}_{\Delta} = 30 + j10 = 31.62\angle 18.43^{\circ}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{173\angle 0^{\circ}}{31.62\angle 18.43^{\circ}} = \underline{\mathbf{5.47\angle -18.43^{\circ}\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^{\circ} = \underline{\mathbf{5.47\angle -138.43^{\circ}\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^{\circ} = \underline{\mathbf{5.47\angle 101.57^{\circ}\ A}}$$

The line currents are

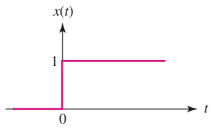
$$\mathbf{I}_{a} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30^{\circ}$$

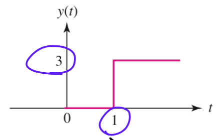
$$\mathbf{I}_{a} = 5.47\sqrt{3}\angle -48.43^{\circ} = \underline{\mathbf{9.474\angle -48.43^{\circ}\ A}}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120^{\circ} = \underline{\mathbf{9.474\angle -168.43^{\circ}\ A}}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120^{\circ} = \underline{\mathbf{9.474\angle 71.57^{\circ}\ A}}$$

Ex: obtain $x(t) * y(t)$





$\therefore\ x(t) = u(t)$; $y(t) = 3u(t-1)$

$$x(t) * y(t) = \int_0^{\infty} x(t-z)\, y(z)\, dz$$

$$= 3\int_0^{\infty} u(t-z)\, u(t-1)\, dz$$